An Accurate Analytical Representation of the Continuous Spectrum Excited on Multilayer Stripline Structures in Spectral-Gap Regions

Paolo Baccarelli, Paolo Burghignoli, Fabrizio Frezza,
Alessandro Galli, Giampiero Lovat, and Simone Paulotto

"La Sapienza" University of Rome, Electronic Engineering Department, Rome, Italy.

***Abstract* — In this work, an approximate asymptotic closed-form representation is derived for the continuous-spectrum current excited by a delta-gap source in a multilayer stripline. The proposed representation is accurate also very close to the source and is valid inside the low-frequency spectral-gap region of the dominant leaky mode supported by this structure, where the main contribution to the continuous-spectrum current is given by the residual-wave current. Numerical results which confirm the accuracy and numerical efficiency of the proposed approach are provided at different frequencies.**

I. INTRODUCTION AND BACKGROUND

The excitation of leaky modes in planar waveguides has received considerable attention in recent years, both from a theoretical and a numerical point of view [1-3]. As is well known, leaky modes may furnish a rapidly convergent representation of the continuous spectrum excited by a finite source [4]: such continuous spectrum may cause undesirable spurious effects, which can give rise to an attenuation of the signal along the line, to interferences with the bound mode, and to cross-talk with neighboring circuits [5].

As shown in [6,7], the stripline structure supports a dominant leaky mode with a very wide spectral gap which begins at zero frequency. Inside the spectral gap the leaky mode is nonphysical and does not provide an accurate representation of the continuous spectrum at all. The part of the continuous spectrum not represented by the (physical) leaky modes has been studied in [8], where it was termed *residual wave*. In [9] a simple asymptotic expression for such residual wave has been derived, which however is not very accurate near the source and inside the above-mentioned spectral-gap region, where the residual wave represents the dominant contribution to the whole continuous spectrum.

In this work, a new improved asymptotic representation for the residual-wave current excited on a multilayer stripline by a delta-gap source is derived in a convenient closed form. The proposed formulation allows to accurately represent the continuous spectrum at low frequencies, also very close to the source region, with considerable computational advantages with respect to a full-wave approach.

This paper is organized as follows. In Section II the derivation of the proposed expression for the continuous spectrum is summarized. In Section III numerical results are given and discussed for a representative two-layer structure. Finally, in Section IV conclusions are presented.

II. ANALYSIS

We consider here a multilayer stripline structure (see the inset of Fig. 1) excited by a delta-gap source with longitudinal profile $L_g(z)$ and transverse profile $\eta(x)$. The current density on the strip is assumed to be $J(x,z)=\eta(x)I(z)z_0$, with a factorized dependence on longitudinal and transverse coordinates. By means of a Galerkin Moment-Method (MoM) in the spectral domain, the spectral current on the strip can be written as [2]:

$$\tilde{I}(k_z)=\frac{2\pi\tilde{L}_g(k_z)}{\int_{-\infty}^{+\infty}\tilde{\eta}^2(k_x)\tilde{G}_{zz}(k_x,k_z)\mathrm{d}k_x} \tag{1}$$

where $\tilde{G}_{zz}(k_x,k_z)$ is the relevant spectral-domain Green's function of the background structure, which is known in a simple closed form.

The integral in the denominator of Eq. (1) is a multivalued function of k_z with branch points equal to the propagation constants k_{pn} $(n=1,2,\ldots)$ of the background-structure modes [2]. The zeroes of the denominator are the solutions of the dispersion equation for the stripline, i.e., they are the propagation constants of the stripline modes; in particular, by choosing the integration path C_T in the complex k_x plane (see Fig. 1), the guided (real *proper*) modes are obtained, while the choice of the integration path C_B furnishes the leaky (complex or real *improper*) modes. Assuming that the only mode above cutoff of the background structure is the TM_0 mode, there is just one branch point $k_z=k_{TM0}$ on the real axis of the k_z plane (all the other branch points are located on the imaginary axis).

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Our aim is to derive an accurate evaluation of the residual-wave current related to such branch point. This is important because in the spectral gap the residual-wave current is the main contribution to the continuous spectrum [9]. The TM_0 residual-wave (RW) current can be evaluated by an integration in the k_z plane along the steepest-descent path (SDP) through the k_{TM0} branch point [9]:

$$I_{RW}(z)=\frac{1}{2\pi}\int_{SDP}\widetilde{I}(k_z)e^{-jk_z z}\mathrm{d}k_z=e^{-jk_{TM0}z}\int_0^{+\infty}F(s)e^{-sz}\mathrm{d}s \quad (2)$$

where $k_z=k_{TM0}-js$ has been used and $F(s)$ is given by:

$$F(s)=-\frac{j}{2\pi}\left[\widetilde{I}(k_z)\Big|_{k_z\in Top}-\widetilde{I}(k_z)\Big|_{k_z\in Bottom}\right]_{k_z=k_{TM0}-js} \quad (3)$$

and where *Top* and *Bottom* are the proper and improper Riemann sheets of the k_z plane. To obtain a closed-form expression for $I_{RW}(z)$, we perform an asymptotic evaluation of the integral in Eq. (2) for large *z*: this entails the representation of $F(s)$ in the limit of $s\to 0$. To this aim, the first step involves the evaluation of the following integrals:

$$\widetilde{D}_{T/B}(k_z)=\int_{C_T/C_B}\widetilde{\eta}^2(k_x)\,\widetilde{G}_{zz}(k_x,k_z)\mathrm{d}k_x \quad (4)$$

By choosing a constant transverse function $\eta(x)=1/w$, its Fourier transform $\widetilde{\eta}(k_x)=\mathrm{Sinc}(k_x w)$ does not allow us to evaluate the integral in Eq. (4) by means of the Cauchy Integral Theorem, because the integrand is not infinitesimal at complex infinity. However, by referring for example to the C_T path in Fig. 1, we may express the *Sinc* function in terms of complex exponentials, thus obtaining:

$$\widetilde{D}_T(k_z)=2\int_{C_T}\frac{1-e^{jk_x w}}{k_x^2 w^2}\,\widetilde{G}_{zz}(k_x,k_z)\mathrm{d}k_x \quad (5)$$

Now we can evaluate the integral in Eq. (5) by means of the closed contour shown in Fig. 2, which encloses an infinite number of poles $k_{x_{pn}}=\sqrt{k_{pn}^2-k_z^2}$ (n = 1,2,...). By letting $r\to 0$ and $R\to\infty$ in Fig. 2, we obtain:

$$\widetilde{D}_T(k_z)=\frac{2\pi}{w}\widetilde{G}_{zz}(0,k_z)+ +4\pi j\frac{1-e^{jk_{x_{TM0}}w}}{k_{x_{TM0}}^2 w^2}\mathrm{Res}\left[\widetilde{G}_{zz}(k_x,k_z)\right]_{k_x=k_{x_{TM0}}}+ +4\pi j\sum_{n=2}^{+\infty}\frac{1-e^{jk_{x_{pn}}w}}{k_{x_{pn}}^2 w^2}\mathrm{Res}\left[\widetilde{G}_{zz}(k_x,k_z)\right]_{k_x=k_{x_{pn}}} \quad (6)$$

where the first-order pole in $k_x=0$, due to the choice of the basis function $\eta(x)$, has given rise to the first addend in the right-hand side of Eq. (6), and the contribution in $k_{x_{p1}}=k_{x_{TM0}}$ has been isolated. As shown in [9], when $k_z\to k_{TM0}$, the $\widetilde{G}_{zz}(k_x,k_z)$ function can approximately be written as a function of $k_t=\sqrt{k_x^2+k_z^2}$; with the change of variable $k_z=k_{TM0}-js$, after some algebra, Eq. (6) (and its analogue for the case of the C_B path) can be written, in the limit of $s\to 0$:

$$\widetilde{D}_{T/B}\cong k_{TM0}\mathrm{Res}\left[\widetilde{G}_{zz}(k_t)\right]_{k_t=k_{TM0}}\cdot \cdot\left(A\pm\frac{2\pi j}{\sqrt{2jk_{TM0}s+s^2}}\pm\frac{\pi w^2}{6j}\sqrt{2jk_{TM0}s+s^2}\right) \quad (7)$$

where A is a complex coefficient whose calculation involves the summation of a series similar to the series occurring in Eq. (6), suitably truncated to a finite number of terms (typically 20-30). Moreover, the plus and minus signs in Eq. (7) correspond to the integrals along the C_T and C_B paths, respectively.

By means of Eq. (7), $F(s)$ can be written, through Eqs. (1) and (3), as:

$$F(s)\cong N_0\sqrt{s}\left(\frac{1}{s-s_1}-\frac{1}{s-s_2}\right) \quad (8)$$

where the presence of two poles s_1 and s_2 can be observed, and N_0 is a suitable complex coefficient. On the basis of Eq. (8), the integral in Eq. (2) can be evaluated in a closed form; the result is [10]:

$$I_{RW}(z)\cong j\pi N_0 e^{-jk_{TM0}z}\cdot \cdot\left\{\sqrt{s_1}\,e^{-s_1 z}\left[\mathrm{Sgn}\,[\Im m[s_1]]+\mathrm{Erf}(j\sqrt{s_1 z})\right]+ -\sqrt{s_2}\,e^{-s_2 z}\left[\mathrm{Sgn}\,[\Im m[s_2]]+\mathrm{Erf}(j\sqrt{s_2 z})\right]\right\} \quad (9)$$

where the principal determination of the square-root function is defined as $-\pi/2\le Arg(\sqrt{\cdot})<\pi/2$, *Sgn* is the sign function, and *Erf* is the error function.

III. NUMERICAL RESULTS

To validate the proposed formulation, we show residual-wave current calculations for a two-layer stripline with parameters h_1 = 1 mm, h_2 = 0.5 mm, w = 7 mm, ε_{r1} = 10, ε_{r2} = 1 (see the inset of Fig. 3). The relevant dispersion curves for the phase constant of both the dominant bound mode (EH_0) and the dominant leaky mode of the stripline are shown in Fig. 3, together with the TM_0 mode of the background structure.

The spectral-gap region corresponds to the frequency range where the leaky pole is not physical [9]. We can divide it into two ranges, depending on the real or complex nature of the involved leaky poles. In the first range there are two *improper real* solutions of the dispersion equation: this region begins at zero frequency and ends at the *splitting-point* frequency, where the two real improper poles coalesce into one double improper pole (point SP in Fig. 3). In the second range there are two *improper complex-conjugate* solutions, corresponding to two nonphysical leaky poles: this region begins at the splitting-point frequency and ends at the *crossing-point* frequency (point CP in Fig. 3). At the crossing point the phase constants of the dominant leaky mode and the TM_0 mode are equal, and therefore *one* of the two leaky poles becomes physical [10].

In the subsequent figures we show a comparison among: *i*) the 'exact' (full wave) RW current, calculated by means of a Galerkin MoM in the spectral domain by using five longitudinal (z-directed) and four transverse (x-directed) basis functions for the current profile, and numerically integrating along the SDP in the spectral k_z plane; *ii*) the asymptotic RW current, calculated according to Eqs. (31) and (32) in [9]; *iii*) the proposed formulation for the RW current, calculated according to Eq. (9) above.

In Fig. 4 the amplitude of the RW current as a function of the normalized longitudinal abscissa z/λ_0 is reported in a logarithmic scale at the frequency $f = 1$ GHz, for which the poles are an improper complex-conjugate pair: as it can be seen, the proposed formulation is in a very good agreement with the exact RW current also very close to the source, while the asymptotic formulation begins to be accurate only for $z \gtrsim 4\ \lambda_0$.

In Fig. 5 the same comparison is shown at $f = 0.5$ GHz, for which the poles are improper real: in this case the asymptotic formulation is substantially inaccurate in the displayed range of z/λ_0 values, while our formulation is still in a very good agreement with the exact RW. This is even more evident in Fig. 6, where the comparison at $f = 0.25$ GHz (for which the poles are again improper real) shows a greater discrepancy between the asymptotic formulation and our formulation. The latter increases its accuracy by lowering the frequency, since one improper real pole tends to the k_{TM0} branch point, and therefore the main contribution to the SDP integral comes from a neighborhood of k_{TM0}, coherently with the limit $s \to 0$ used to derive the proposed representation.

Finally, it is worth noting that the implementation of our formulation requires about half a minute of CPU time on a standard PC, while the full-wave approach requires hours of calculation.

IV. CONCLUSION

In this work, an original closed-form representation of the residual-wave current excited by a delta-gap source in a multilayer stripline has been presented. The proposed formulation allows us to evaluate in a simple and analytical form the continuous-spectrum current in the spectral-gap region of the dominant leaky mode, which is very wide especially for narrow metal strips. Such continuous-spectrum current may adversely affect the performance of a given structure, due to signal degradation and interference with neighboring circuits. The proposed formulation furnishes an accurate representation of the residual-wave current also very near the source, and it is computationally very convenient if compared with a full-wave approach.

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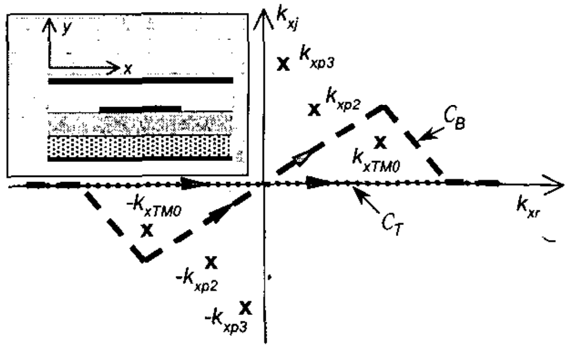

Fig. 1 – Integration paths C_B and C_T in the k_x plane, which give rise to leaky and bound modes, respectively; crosses represent the spectral Green's function pole locations. *Inset*: transverse section of the multilayer stripline structure.

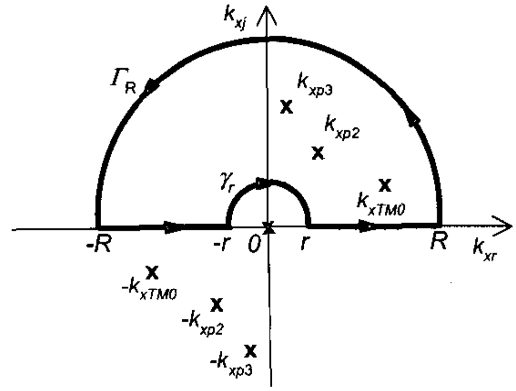

Fig. 2 – Integration contour used to apply Cauchy Integral Theorem for the evaluation of the integral function $\tilde{D}_T(k_z)$.

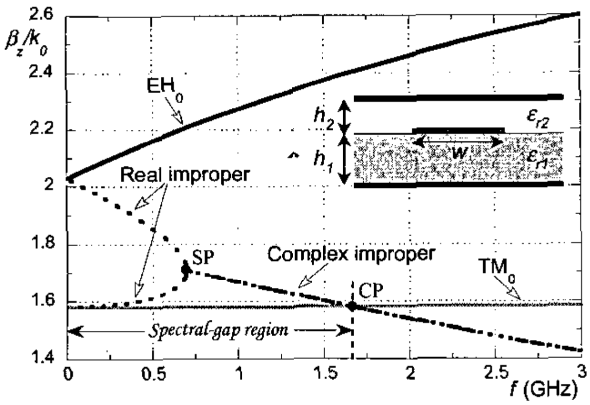

Fig. 3 – Normalized phase constant (β/k_0) vs. frequency f for the fundamental guided mode EH_0 of the stripline (*solid line*), the dominant leaky mode of the stripline (*dotted line*: real improper; *broken line*: complex improper), and the TM_0 mode of the background structure (*light solid line*). *Parameters*: h_1 = 1 mm, h_2 = 0.5 mm, w = 7 mm, ε_{r1} = 10, ε_{r2} = 1 (see the inset).

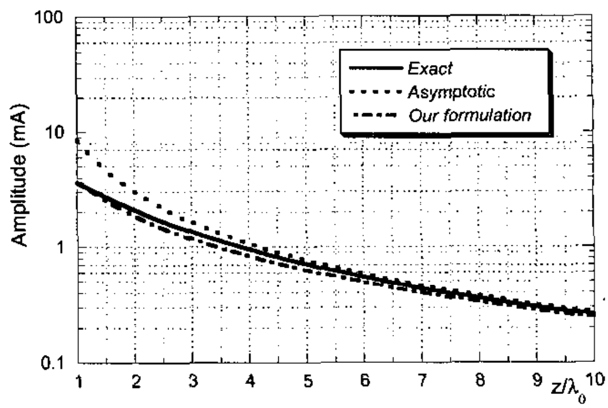

Fig. 4 – Amplitude of the residual-wave current for a structure as in Fig. 3, as a function of the normalized longitudinal abscissa z/λ_0, at the frequency f = 1 GHz (for reference, the current amplitude of the EH_0 guided mode is about 50 mA).

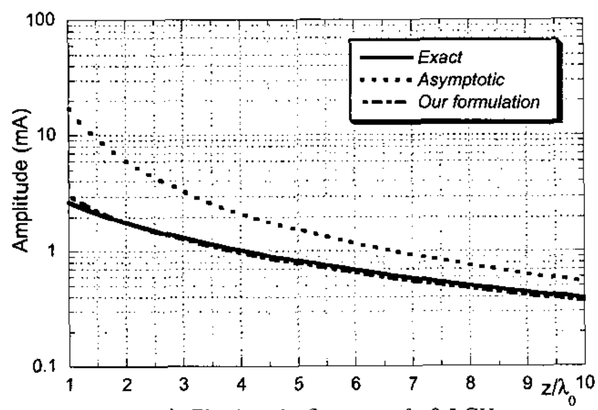

Fig. 5 – Same as in Fig. 4, at the frequency f = 0.5 GHz.

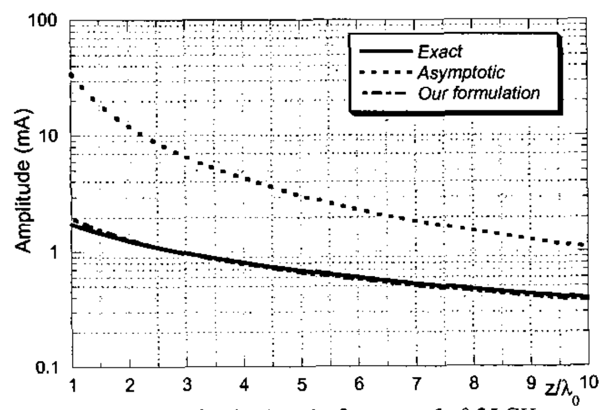

Fig. 6 – Same as in Fig. 4, at the frequency f = 0.25 GHz.